Vanishing results for the Aomoto complex of real hyperplane arrangements via minimality

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Mike Falk's 60th birthday, Stony Brook AMS Meeting, 19-20 March 2016

P. Bailet, M. Yoshinaga: *Vanishing results for the Aomoto complex of real hyperplane arrangements via minimality; arXiv:1512.05318*

The objects

Let $\mathcal{A} = \{H_1, \ldots, H_n\} \subset \mathbb{R}^\ell$ be an affine hyperplane arrangement

$M(\mathcal{A}) = \mathbb{C}^\ell \setminus \bigcup_{i=1}^n H_i \otimes \mathbb{C}$: the complement of the complexified hyperplane, which is a **minimal space** (Dimca, Papadima, Randell)

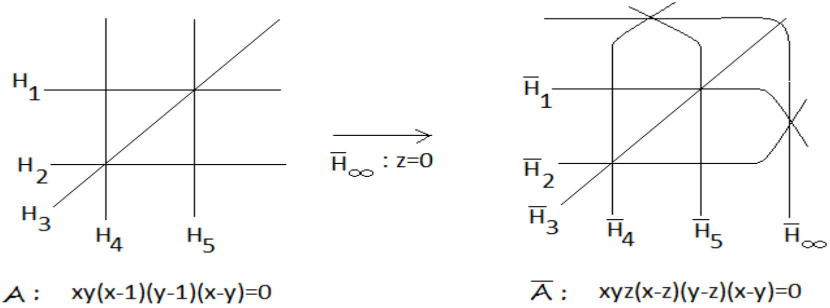
$L(\mathcal{A})$: intersection lattice of $\mathcal{A}$

By identifying

$$\mathbb{R}^\ell \simeq \mathbb{P}^\ell_{\mathbb{R}} \backslash \overline{H}_\infty,$$

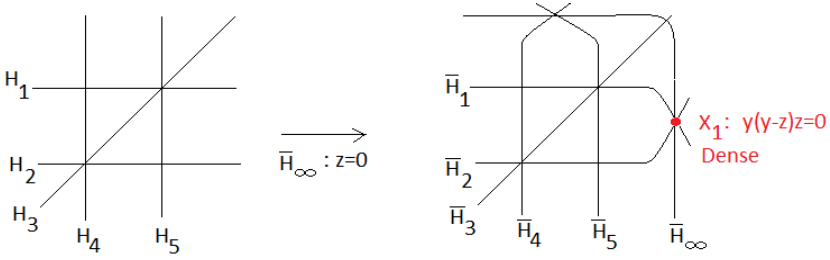
we consider the **projective closure of** $\mathcal{A}$:

$$\overline{\mathcal{A}} = \{\overline{H}_1, \dots, \overline{H}_n, \overline{H}_\infty\} \subset \mathbb{P}^\ell_{\mathbb{R}}.$$



Definition

*An edge $X \in L(\overline{\mathcal{A}})$ is **dense** if $\overline{\mathcal{A}}_X = \{\overline{H}_i \in \overline{\mathcal{A}} \mid X \subseteq \overline{H}_i\}$ is indecomposable.*



$\mathcal{A}$: xy(x-1)(y-1)(x-y)=0

$\overline{\mathcal{A}}$: xyz(x-z)(y-z)(x-y)=0

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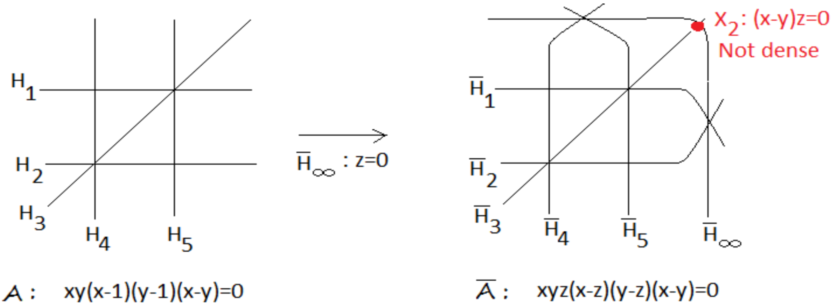

$\mathcal{A} = \{H_1, \dots, H_n\} \subset \mathbb{R}^\ell$, $H_i \leftrightsquigarrow e_i$.

Definition

Let R be a commutative unitary ring. The ***Orlik-Solomon algebra*** *$A_R^\bullet(\mathcal{A})$ of $\mathcal{A}$ is the quotient of the exterior algebra generated by the elements e_i, $1 \le i \le n$, modulo the ideal $I(\mathcal{A})$ generated by:*

- *the elements of the form*

$$\{e_{i_1} \wedge \cdots \wedge e_{i_s} \mid H_{i_1} \cap \cdots \cap H_{i_s} = \emptyset\},$$

- *the elements of the form*

$$\{\partial(e_{i_1} \wedge \cdots \wedge e_{i_s}) \mid H_{i_1} \cap \cdots \cap H_{i_s} \neq \emptyset,\ \mathrm{codim}(H_{i_1} \cap \cdots \cap H_{i_s}) < s\},$$

where
$\partial(e_{i_1} \wedge \cdots \wedge e_{i_s}) = \sum_{\alpha=1}^{s} (-1)^{\alpha-1} e_{i_1} \wedge \cdots \wedge \widehat{e_{i_\alpha}} \wedge \cdots \wedge e_{i_s}$.

(Orlik-Solomon,1980)

$$A^\bullet_{\mathbb{Z}}(\mathcal{A}) \simeq H^\bullet(M(\mathcal{A}),\mathbb{Z}),\ e_i \simeq \frac{1}{2\pi\sqrt{-1}} d\log\alpha_i,$$

where $H_i:\ \alpha_i = 0$.

Let $\lambda = (\lambda_1,\dots,\lambda_n) \in R^n$ and $\omega_\lambda = \sum_{i=1}^n \lambda_i e_i \in A^1_R(\mathcal{A})$.
The **Aomoto complex** is the cochain complex

$$(A^\bullet_R(\mathcal{A}),\omega_\lambda\wedge) = \{A^\bullet_R(\mathcal{A}) \xrightarrow{\omega_\lambda\wedge} A^{\bullet+1}_R(\mathcal{A})\}.$$

QUESTION:

$$H^{\bullet}(A_R^{\bullet}(\mathcal{A}), \omega_\lambda \wedge)?$$

Vanishing results for $H^{\bullet}(A_R^{\bullet}(\mathcal{A}), \omega_\lambda \wedge)$?

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Related with:

- **Resonance varieties** (Falk)
- **Local system cohomology of the complement** $H^\bullet(M(\mathcal{A}), \mathcal{L})$ (Tangent cone theorem, upper bound for torsion local system of Suciu and Papadima 2010)

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We have:

- Vanishing results for $H^\bullet(A^\bullet_R(\mathcal{A}), \omega_\lambda \wedge)$ (Yuzvinsky 1995, 2001)
- Vanishing results for $H^\bullet(M(\mathcal{A}), \mathcal{L})$ (Cohen-Dimca-Orlik 2003)

A vanishing result

Set $\lambda_\infty := -\sum_{i=1}^{n} \lambda_i,$ and for any $X \in L(\overline{\mathcal{A}})$,

$$\lambda_X := \sum_{\overline{H}_i \supset X} \lambda_i, \text{ where } i \text{ runs } \{1, 2, \ldots, n, \infty\}.$$

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Theorem (B., Yoshinaga)

Assume $\mathcal{A} = \{H_1, \dots, H_n\}$ is essential and $\lambda_X \in R^\times$ for any dense edge X contained in the hyperplane at infinity $\overline{H}_\infty$. Then the following holds:

$$H^k(A_R^\bullet(\mathcal{A}), \omega_\lambda \wedge) \simeq \begin{cases} 0, & \text{if } k \neq \ell, \\ R^{|\chi(M(\mathcal{A}))|}, & \text{if } k = \ell. \end{cases}$$

Corollary

Let $0 \leq p < \ell$*. If* $\lambda_X \in R^\times$ *for for any dense edge* $X \in L(\overline{\mathcal{A}})$ *contained in the hyperplane at infinity with* $\dim(X) \geq p$*, then*

$$H^k(A_R^\bullet(\mathcal{A}), \omega_\lambda \wedge) \simeq 0, \text{ for all } 0 \leq k < \ell - p.$$

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Theorem (Cohen-Dimca-Orlik, 2003)

Let $\mathcal{A} = \{H_1, \dots, H_n\} \subset \mathbb{C}^\ell$ be a complex hyperplane arrangement. Suppose that $q_X \neq 1$ for each dense edge X contained in the hyperplane at infinity. Then the following holds:

$$\dim H^k(M(\mathcal{A}), \mathcal{L}) \simeq \begin{cases} 0, & \text{if } k \neq \ell, \\ |\chi(M(\mathcal{A}))|, & \text{if } k = \ell. \end{cases}$$

Description of the minimal complex in terms of real structures.

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Proposition (Yoshinaga, 2007)

There exists an isomorphism of cochain complexes

$$(R[\mathrm{ch}^\bullet(\mathcal{A})], \nabla_{\omega_\lambda}) \simeq (A_R^\bullet(\mathcal{A}), \omega_\lambda \wedge).$$

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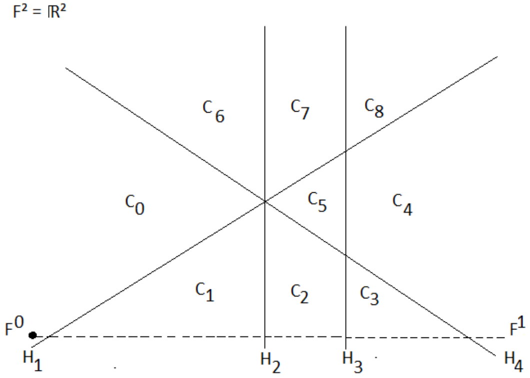
$$\mathcal{F} : \emptyset = F^{-1} \subset F^0 \subset F^1 \subset \cdots \subset F^\ell = \mathbb{R}^\ell$$

and define

$$\begin{aligned} \mathrm{ch}^k(\mathcal{A}) &= \{C \in \mathrm{ch}(\mathcal{A}) \mid C \cap F^k \neq \emptyset, C \cap F^{k-1} = \emptyset\} \\ \mathrm{bch}^k(\mathcal{A}) &= \{C \in \mathrm{ch}^k(\mathcal{A}) \mid C \cap F^k \text{ is bounded}\} \\ \mathrm{uch}^k(\mathcal{A}) &= \{C \in \mathrm{ch}^k(\mathcal{A}) \mid C \cap F^k \text{ is unbounded}\}. \end{aligned}$$



$ch^0 = \{C_0\} = bch^0$

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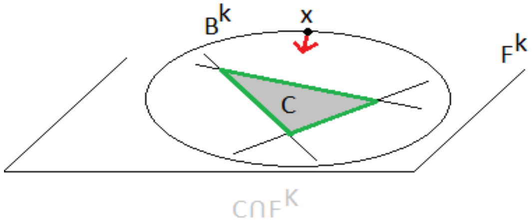
Definition

Given $C \in \mathrm{ch}^k(\mathcal{A})$ and $C' \in \mathrm{ch}^{k+1}(\mathcal{A})$, the degree $\deg(C, C')$ between C and C' is

$$\deg(C, C') := \deg\left(\left.\frac{U^{C'}}{|U^{C'}|}\right|_{\partial(\overline{C}\cap B^k)} : \partial(\overline{C} \cap B^k) \longrightarrow S^{k-1} \right) \in \mathbb{Z}$$

where $U^{C'} \in TF^k$ is a vector field directed to C' (independent of the choice of $U^{C'}$).



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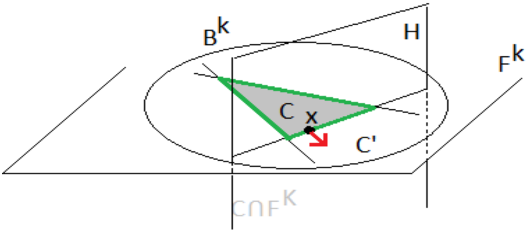

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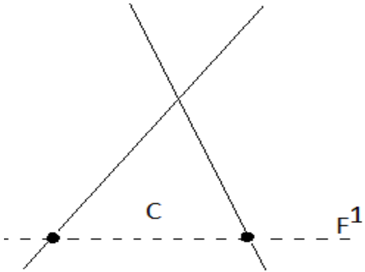

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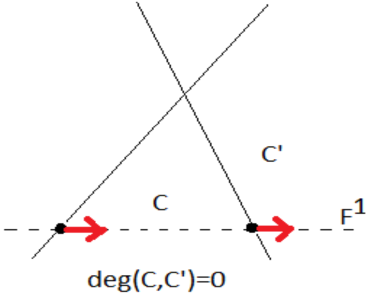

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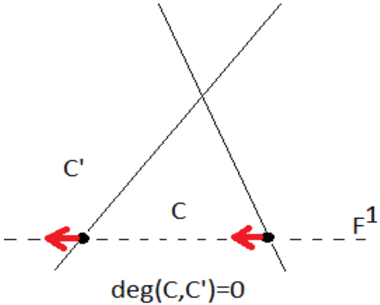

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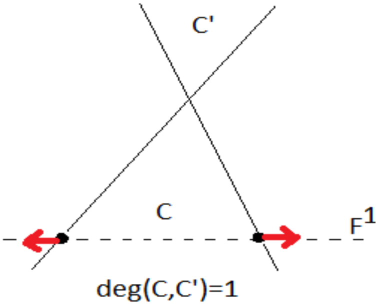

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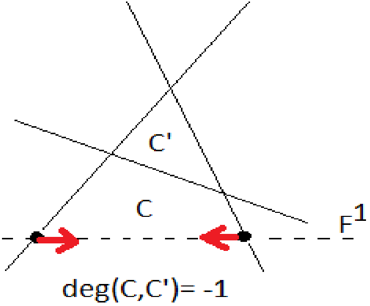
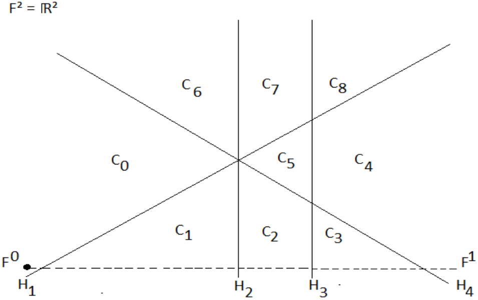


- $k = 0 : \deg(C, C') = 1$.
- $k = 1 : \deg(C, C')$ is easy to compute.
- $k > 1 : \deg(C, C')$ is difficult.

Formula for the coboundary map:

$$\begin{aligned}\nabla_{\omega_\lambda} : R[\mathrm{ch}^k(\mathcal{A})] &\longrightarrow R[\mathrm{ch}^{k+1}(\mathcal{A})] \\ [C] &\longmapsto \sum_{C' \in \mathrm{ch}^{k+1}} \deg(C, C') \cdot \lambda_{\mathrm{Sep}(C,C')} \cdot [C'],\end{aligned}$$

where $\lambda_{\mathrm{Sep}(C,C')} = \sum_{\substack{H_i \text{separates} \\ C \text{ and } C'}} \lambda_i.$



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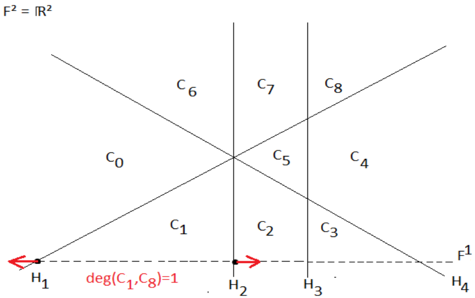
$$\nabla_{\omega_\lambda} : R[\mathrm{ch}^1(\mathcal{A})] \longrightarrow R[\mathrm{ch}^2(\mathcal{A})].$$

$$\begin{array}{cc} & \begin{array}{cccc} \nabla_{\omega_\lambda}([C_1]) & \nabla_{\omega_\lambda}([C_3]) & \nabla_{\omega_\lambda}([C_2]) & \nabla_{\omega_\lambda}([C_4]) \end{array} \\ \begin{array}{c} C_8 \\ C_6 \\ C_7 \\ C_5 \end{array} & \left(\begin{array}{cccc} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{array} \right) \end{array}$$



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$$\nabla_{\omega_\lambda}([C]) = \sum_{C' \in \mathrm{ch}^2} \deg(C, C') \cdot \lambda_{\mathrm{Sep}(C,C')} \cdot [C'].$$

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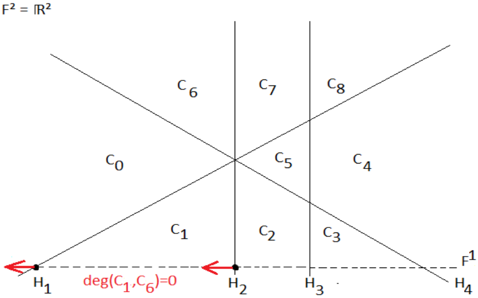

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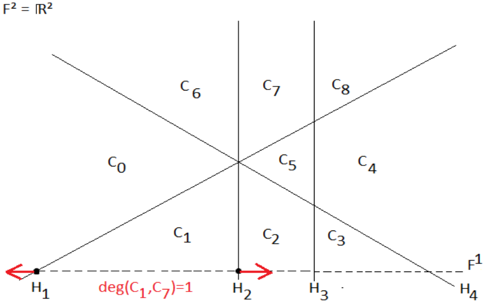

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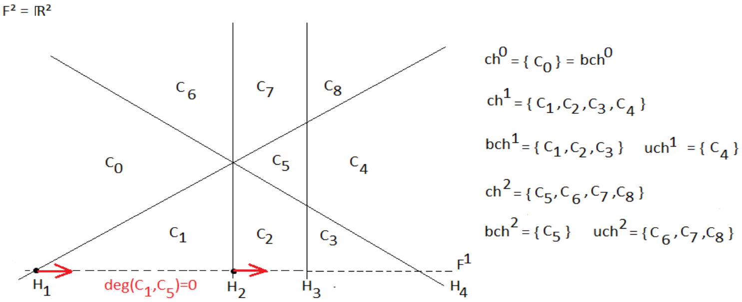

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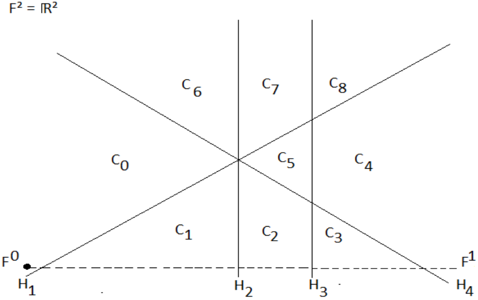

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\end{array}\\
\begin{pmatrix}
1\times(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & 0 & 0 & -\lambda_1 \\
0 & \lambda_1+\lambda_2+\lambda_3+\lambda_4 & 0 & -(\lambda_1+\lambda_2+\lambda_3) \\
1\times(\lambda_1+\lambda_2+\lambda_4) & \lambda_1+\lambda_3+\lambda_4 & -(\lambda_1+\lambda_4) & -(\lambda_1+\lambda_3) \\
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$$

$$\mathrm{bch}^1$$

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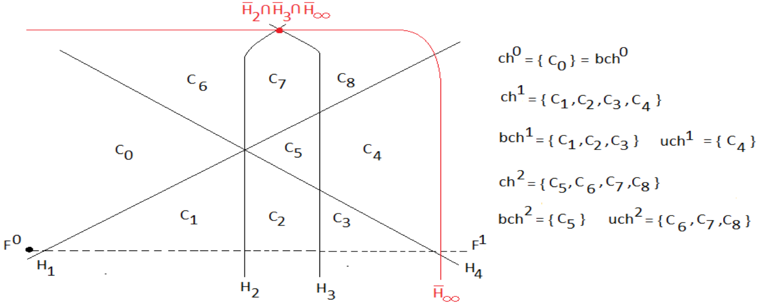
\end{array}$$

$$\overline{\mathrm{M}}$$

- $\overline{M} = Mat(\overline{\nabla}_{\omega_\lambda} : R[\mathrm{bch}^1(\mathcal{A})] \longrightarrow R[\mathrm{uch}^2(\mathcal{A})])$ is triangular
- $\det(\overline{M}) = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)^2(\lambda_1 + \lambda_4)$
  $= \lambda_\infty^2(\lambda_\infty + \lambda_2 + \lambda_3)$

$$(\lambda_\infty := -\sum_{i=1}^{n} \lambda_i,\ \lambda_X := \sum_{\overline{H}_i \supset X} \lambda_i)$$



- $\overline{M} = Mat(\overline{\nabla}_{\omega_\lambda} : R[\mathrm{bch}^1(\mathcal{A})] \longrightarrow R[\mathrm{uch}^2(\mathcal{A})])$ is triangular
- $\det(\overline{M}) = \lambda^2_{\overline{H}_\infty} \lambda_{\overline{H}_2 \cap \overline{H}_3 \cap \overline{H}_\infty}$, where $\overline{H}_\infty$ and $\overline{H}_2 \cap \overline{H}_3 \cap \overline{H}_\infty$ are the dense edges of $L(\overline{\mathcal{A}})$ contained in the hyperplane at infitnity $\overline{H}_\infty$.

In order to study the $H^k(A_R^\bullet(\mathcal{A}), \omega_\lambda \wedge) \simeq H^k(R[\mathrm{ch}^\bullet(\mathcal{A})], \nabla_{\omega_\lambda})$, we consider

$$\overline{\nabla}_{\omega_\lambda} : R[\mathrm{bch}^k(\mathcal{A})] \longrightarrow R[\mathrm{uch}^{k+1}(\mathcal{A})].$$

Choosing a good ordering of the chambers $\mathrm{bch}^k(\mathcal{A})$, one can show that:

- $\overline{M} = Mat(\overline{\nabla}_{\omega_\lambda})$ is triangular.
- $\det(\overline{M}) = \pm \prod_{\substack{X \in L(\overline{\mathcal{A}}) \text{ dense} \\ X \subseteq \overline{H}_\infty}} \lambda_X^{n_X}$, where $n_X \in \mathbb{Z}_{>1}$.

It follows that if the assumptions of our theorem are satisfied ($\lambda_X \in R^\times, \forall X \in L(\mathcal{A})$ dense, $X \subseteq \overline{H}_\infty$) then:

$$\overline{\nabla}_{\omega_\lambda} \text{ is an isomorphism}$$
$$\Downarrow$$
$$H^k(A_R^\bullet(\mathcal{A}), \omega_\lambda \wedge) \simeq 0, \ \forall k \neq \ell.$$

$$\overline{\nabla}_{\omega_\lambda} \text{ is an isomorphism}$$
$$\Downarrow$$
$$H^k(A^\bullet_R(\mathcal{A}), \omega_\lambda \wedge) \simeq 0, \ \forall k \neq \ell.$$

Thank you very much for your attention and...
happy birthday Mike!!!